

A Dual-Wavelength Radar Method to Measure Snowfall Rate

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ABSTRACT

A dual-wavelength radar method to estimate snowfall rate has been developed. The method suggests taking simultaneous and collocated reflectivity measurements at two radar wavelengths. Snowfall backscattering at one of these wavelengths should be in the Rayleigh regime or sufficiently close to this regime, while backscattering at the other wavelength should be substantially outside this regime for typical snowflake sizes. Combinations of K_a-band (for a shorter wavelength) and X-, C-, or S-band (for a longer wavelength) radar measurements satisfy this requirement. The logarithmic difference between reflectivities at these two wavelengths provides an independent estimate of snowflake median size D_m , which exhibits a very low sensitivity to snowflake density and details of the size distribution. The estimates of D_m and radar reflectivities Z_e at the longer wavelength are then used to obtain snowfall rate R from the Z_e - R - D_m relationships, which have a snowflake effective density ρ_e as a "tuning" parameter. The independent information about snowflake characteristic size accounts for much of the improvement of the dual-wavelength method over traditional, single-parameter Z_e - R relationships.

The paper also presents experimental data collected during January–March 1996, near Boulder, Colorado, with the National Oceanic and Atmospheric Administration's K_a- and X-band radars. The radar data were supplemented by simultaneous ground measurements of snow accumulation. Comparisons of the ground and dual-wavelength radar measurements indicate that a tuning value ρ_e of about 0.03–0.04 g cm⁻³ provides a good match with surface-observed snow accumulations. Differences in dual-wavelength radar estimates of accumulation for ρ_e between 0.03 and 0.04 g cm⁻³ are usually within 25%, while existing X-band, single-parameter Z_e - R relationships yield accumulations that differ by as much as a factor of 4.

1. Introduction

Estimation of precipitation rate and accumulation is one of the most important tasks of radar meteorology. Traditional, single-parameter radar methods to estimate precipitation rate R from measurements of equivalent radar reflectivity, Z_e , rely on so-called Z_e - R relationships that are derived in the power-law form. Such relationships are derived either from simultaneous radar measurements and data of precipitation gauges or from calculations of both Z_e and R from model or measured precipitation-particle-size spectra (assuming a certain relation between particle fall velocities and their sizes).

The Z_e - R relationships represent an average correlation between Z_e and R because both of these parameters depend differently on functions of precipitation-particle-size and velocity distributions. The density and shape of precipitation particles also influence these relationships. Even for a simpler case of rain, these relationships depend on type of rain. Estimates of snowfall

using Z_e - R relationships are often significantly less accurate than for rainfall. This can be explained by a much greater variety of shapes, densities, and terminal fall velocities (as a function of snowflake size) of snowflakes compared to those of raindrops. In addition, the low correlation between characteristic size of snowflakes and their concentrations—two major parameters determining both R and Z_e —makes single-parameter radar measurements of snowfall rate generally insufficient.

The problems described above result in a very high variability in the coefficients of Z_e - R power-law relationships for snowfall. This variability is higher than for the rainfall Z_e - R relationships and can be illustrated by several existing relationships at X band ($\lambda \approx 3.2$ cm) derived by different authors: $Z_e = 1050R^2$ (Puhakka 1975), $Z_e = 229R^{1.65}$ (Boucher and Wieler 1985), and $Z_e = 427R^{1.09}$ (Fujiyoshi et al. 1990). In these relationships, as usual, reflectivity, Z_e , is in mm⁶ m⁻³ and the snowfall precipitation rate, R , is given in terms of melted water equivalent in millimeters per hour. Note that for meteorological radars, Z_e is usually calculated assuming the complex refractive index of water. These three relationships were obtained for dry snowfall events from the analysis of radar data and simultaneous measure-

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ments of R by wind-shielded precipitation gauges and/or directly from flat measuring surfaces. A similar high variability in the power-law coefficients is also observed at other radar wavelengths. The X-band relationships are given for this illustration because this wavelength is used as the longer wavelength in the method proposed in this paper. So single-parameter, X-band snowfall estimates were available in addition to dual-wavelength estimates for comparisons.

Different polarization approaches developed to improve radar measurements of rainfall (such as differential reflectivity measurements to get an independent estimate of raindrop characteristic size) are of limited use for snowfall measurements because larger, low-density, irregular-shaped aggregate snowflakes usually exhibit weak polarization signatures, although single, pristine ice crystals often found aloft could produce very distinct polarization patterns (Matrosov et al. 1996). However, the aggregate type of snowflakes usually contributes most to the ground snow accumulation. The snowfall reported here consisted mostly of irregular-type aggregates with typical sizes of a few millimeters.

In this paper, a dual-wavelength radar method is proposed to measure snowfall precipitation rate. This method resolves an ambiguity between contributions of snowflake characteristic size and concentrations in both Z_e and R by having an independent estimate of characteristic size from logarithmic difference of reflectivities at two radar wavelengths. This leads to an improvement of snowfall estimates compared to single-wavelength measurements. Unlike different dual-wavelength attenuation techniques proposed for rainfall-rate estimations, the method suggested here is based on the difference in the backscattering regimes between two different radar wavelengths. This approach is applicable to dry light to moderate snowfalls where attenuation of radar signals is relatively weak. As for single-parameter radar measurements, the absolute radar calibration is very important for the dual-wavelength approach.

2. Snowflake-size spectra

As for raindrops, size distributions of snowflakes are usually modeled by the gamma function of different orders n in terms of diameters of equal-volume spheres D :

$$N(D) = N_o D^n \exp(-\Lambda D), \quad (1)$$

where the parameter Λ is related to the snowflake characteristic size. Most often, the exponential distribution is assumed ($n = 0$), as in classic works by Gunn and Marshall (1958) and Sekhon and Srivastava (1970).

In contrast to raindrops, relatively few in situ measurements of snowflake-size spectra have been published. Braham (1990) reports a rather detailed study of in situ snowflake spectra measured with Particle Measuring Systems (PMS) probes. The reported data contained 49 experimental spectra that were obtained near the ground, well removed from snow-growth regions.

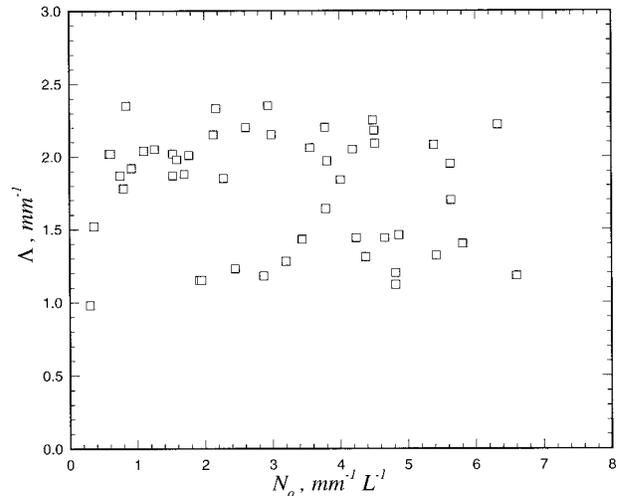


FIG. 1. Relation between parameters N_o and Λ in snowflake-size spectra.

These data indicated that an exponential distribution usually satisfactorily describes experimental spectra.

Figure 1 shows a scatterplot of experimentally derived distribution parameters N_o and Λ , which were plotted here using the data, published as a table, by Braham (1990). In Fig. 1 there is almost no correlation between these parameters. The corresponding linear correlation coefficient is -0.16 . This weak correlation indicates that snowflake-size spectra are described by at least two independent parameters, and one parameter measurement (i.e., Z_e) of snowfall is generally insufficient. Note that the choice of two parameters describing hydrometeor size distribution can be different (Atlas and Chmela 1957). For example, particle characteristic size (e.g., median diameter, D_m) and total snowflake concentration could be used instead of N_o and Λ .

3. Dual-wavelength ratio as an indicator of snowflake characteristic size

a. Choice of wavelengths

It was shown by Matrosov (1992) and Illingworth et al. (1995) that characteristic particle size can be inferred from the logarithmic difference between reflectivities measured at two different radar wavelengths. One wavelength from this pair should provide the Rayleigh regime of scattering for snowflakes, or, at least, deviations from this scattering regime should not be significant. The wavelengths in S band ($\lambda \sim 10$ – 11 cm) are usually long enough to satisfy the Rayleigh conditions ($\pi D/\lambda \ll 1$, and $\pi|m|D/\lambda \ll 1$, where m is the snowflake refractive index) for most snowflakes. Scattering at C band ($\lambda \sim 5$ – 6 cm) and X band ($\lambda \sim 3$ cm) is not already in the Rayleigh regime, but it is still close to this regime in terms of the monotonic increase of backscatter efficiency with snowflake size for typical snowflakes.

Scattering at the other wavelength in the pair should

be sufficiently outside the Rayleigh regime. The K_a -band radar wavelengths ($\lambda \sim 8\text{--}9$ mm) generally satisfy this condition. Scattering at the W band ($\lambda \sim 3$ mm) for typical snowflakes is strongly non-Rayleigh. However, hydrometeor and gaseous attenuation poses a significant problem for this band.

The National Oceanic and Atmospheric Administration (NOAA) Environmental Technology Laboratory (ETL) possesses two transportable radars for the atmospheric research that satisfy the wavelength separation requirement. They transmit at $\lambda = 3.2$ cm (X band) and $\lambda = 8.6$ mm (K_a band), respectively. For most snowfall, scattering regimes at these wavelengths are quite different (Matrosov 1992). Theoretical calculations for this paper were performed for this pair of available radar wavelengths. Collocated, synchronous measurements of snowfall were made with these two radars in a number of field experiments, including the Winter Icing and Storms Project (WISP) in 1991 and the Snowrad experiment described in section 5. The WISP data were ETL's first experimental data obtained for snowflake sizing (Matrosov 1992).

b. Dual-wavelength ratio for spherical particles

The characteristic size of a snowflake population can be defined in several different ways. The most widely used characteristic sizes are mean (D_{mean}), median volume (D_m), and modal (D_{mod}). The definition of D_{mean} is the normalized D -weighted integral of the distribution function (1). The mean (D_m) and modal (D_{mod}) are defined as the size that splits the distribution into two equal-volume parts and the size at which the distribution function reaches a maximum value, respectively. For the distribution given by (1) these sizes can be expressed in terms of D^* :

$$D_m = D^*/(n + 3.67), \quad (2a)$$

$$D_{\text{mod}} = D^*n \quad (n \geq 0), \quad (2b)$$

where $D^* = \Lambda^{-1}$, which also can be considered a characteristic size.

Figures 2a–d show results of model calculations for the dual-wavelength reflectivity difference (in a logarithmic sense) $\Delta Z = Z_e(3.2 \text{ cm}) - Z_e(0.86 \text{ cm})$ as a function of different snowflake characteristic sizes mentioned above. Note that ΔZ is often called the dual-wavelength ratio (DWR, hereafter). The results are shown for three different densities of snowflakes that are characteristic for dry snow— $\rho = 0.03, 0.05,$ and 0.07 g cm^{-3} (Ihara et al. 1982; Magono and Nakamura 1965)—and for different orders of the gamma-function size distributions (n).

As shown in Fig. 2, DWR as a function of any characteristic size exhibits a very low sensitivity to the density of snowflakes if $\text{DWR} \leq 15$ dB. The decrease of DWR due to increasing ρ for the considered densities does not exceed a few tenths of 1 dB. Note that this

result also holds if the density of snowflakes is assumed to be decreasing with size in this density range ($0.07\text{--}0.03 \text{ g cm}^{-3}$) rather than being constant. DWR is also quite insensitive to the details of the size distribution (i.e., the order of the gamma-function size distribution n) when DWR is expressed as a function of D_m (Fig. 2a). However, there is a significant sensitivity to n when DWR is expressed as a function of other characteristic sizes (Figs. 2b–d).

For the dual-wavelength ratio $\text{DWR} \leq 15$ dB, DWR as a function of D_m can be approximated by a power law

$$\text{DWR} \approx 58D_m^{1.66}, \quad (3)$$

where DWR is in decibels and D_m is in centimeters. This approximation gives a simple means to estimate median snowflake sizes from dual-wavelength measurements, and it is in an agreement with previously obtained experimental data (Matrosov 1992).

For most common snowflake sizes of several millimeters, the expected values of DWR are about 5–10 dB, which is easily measured. Such values for snowfall are in contrast with ones for nonprecipitating high-altitude ice clouds, where much smaller characteristic particle sizes usually produce reflectivity differences too small for reliable measurements (Matrosov 1993).

c. Effects of nonsphericity of snowflakes

Atlas et al. (1953) showed that if particle sizes are in the Rayleigh scattering regime, irregular low-density (i.e., optically “soft”) snowflakes backscatter the radiation as spheres with the same mass. “Softness” of particles means that their refractive indices are close to 1. This also results in low depolarization. So it is relatively unimportant what polarization of transmitted radar signals is used (horizontal, vertical, or circular).

To assess the effects of nonsphericity for larger particles, calculations of DWR as a function of different characteristic sizes were performed for snowflakes modeled as oblate spheroids with aspect ratios of 0.6 and 0.3. The calculations in the Mie scattering regime were performed using the T-matrix approach (Barber and Yeh 1975). The results of these calculations for $\text{DWR} \leq 15$ dB (not shown here) were very close to those of spherical particles shown in Fig. 2. So it was assumed that the spherical model for snowflakes can be used for the dual-wavelength approach.

d. Effects of attenuation

Attenuation of radar signals in dry snow is small. Battan (1973) gives the following attenuation formula that is valid in the Rayleigh region:

$$\alpha(\text{dB/km}) = 0.035R^2\lambda^{-4} + 0.0022R\lambda^{-1}, \quad (4)$$

where R is in millimeters per hour and λ is in centimeters. The first term in (4) describes volume scattering,

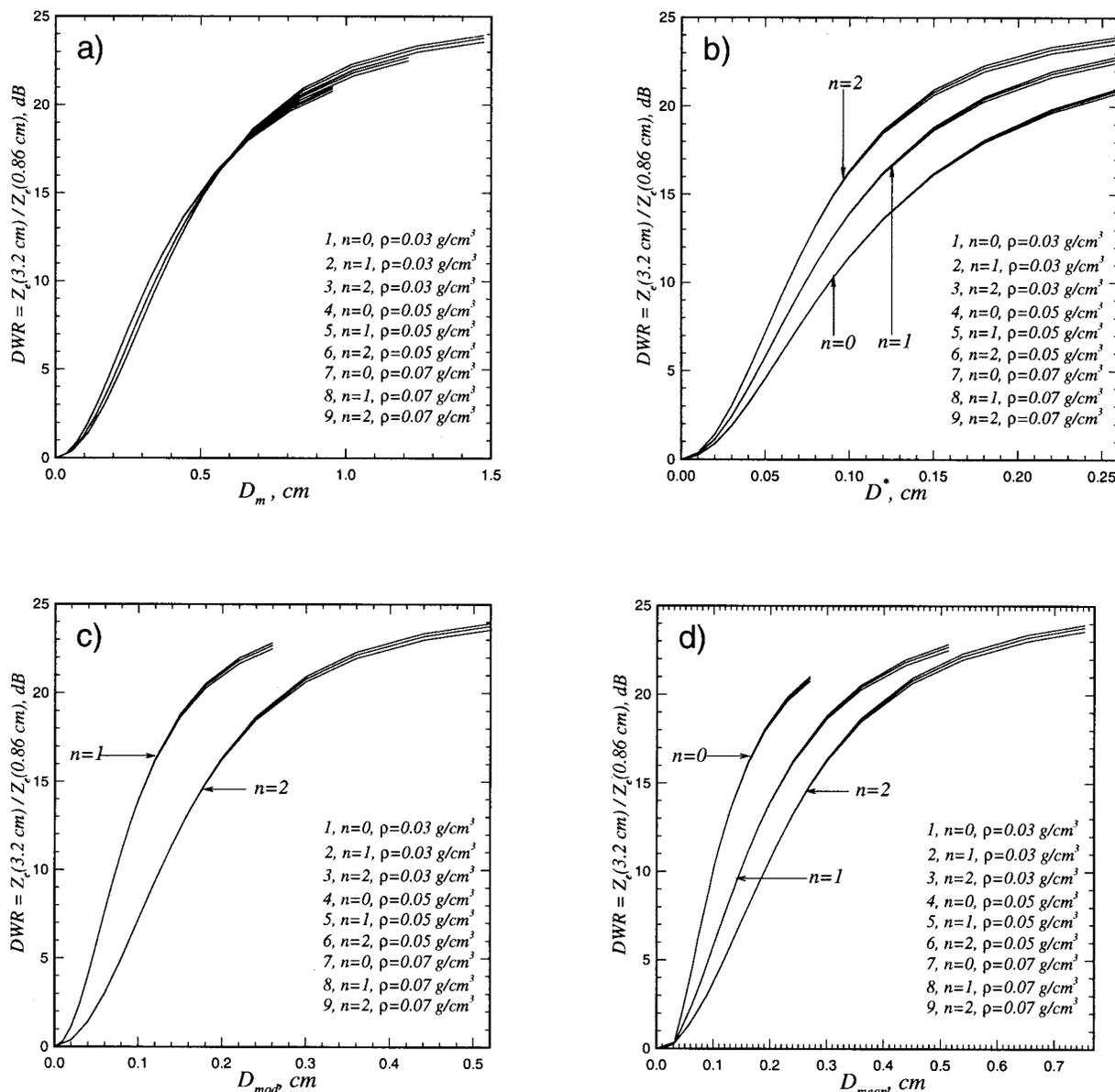


FIG. 2. Dual-wavelength ratio as a function of (a) snowflake median size D_m , (b) D^* , (c) modal size D_{mod} , and (d) mean size D_{mean} for spherical particles of different densities ρ and orders of the gamma-function size distributions (n).

and the second term describes absorption. Equation (4) overestimates attenuation because this formula was derived using the assumption of the particle density of 1 g cm^{-3} for the 0°C temperature (i.e., for relatively high values of the imaginary part of the complex refractive index). Mie calculations for $\lambda = 0.86 \text{ cm}$ and $\lambda = 3.2 \text{ cm}$ showed that α for dry snow at -10°C with typical bulk density of 0.04 g cm^{-3} is about 0.03 and 0.001 dB km^{-1} , respectively, if $R \approx 2 \text{ mm h}^{-1}$ which is a significant intensity for snowfall. These estimates indicate that dry snow attenuation effects can be rather safely ignored for light and moderate snowfalls for not very long ranges. Note that attenuation by atmospheric water

vapor and oxygen (especially for the K_a band) should be taken into account for longer ranges. For winter conditions, this attenuation at the ground level can reach about 0.05 and 0.01 dB km^{-1} for K_a - and X-band wavelengths, respectively.

4. The Z_e - R - D_m relationships for estimating snowfall rate

a. Origin of Z_e - R - D_m relationships

The reflectivity at the longer wavelengths (3.2 cm) can be used to get an estimate of R after snowflake

median size is obtained from DWR measurements. Snowfall rate R is given by the integral

$$R = \frac{\pi}{6} \int_0^{D_{\max}} N(D)v(D)\rho(D)D^3 dD, \quad (5)$$

where $v(D)$ is the snowflake fall velocity as a function of its size.

In contrast to raindrops, fall velocities for snowflakes depend not only on their size but also on snowflake densities and shapes (habits), which vary significantly. A number of fall velocity–size relationships for snowflakes can be found in the literature. A relationship given by Magono and Nakamura (1965),

$$v(D) = 880[(\rho - \rho_a)D]^{0.5}, \quad (6)$$

has been used in many different studies. In (6), ρ and ρ_a are snowflake and air densities, respectively, and the units are in cgs. One of the most recent relationships for aggregate snowflakes can be obtained from the data presented by Mitchell (1996):

$$v(D) = 211D^{0.416}. \quad (7)$$

Both Z_e and R are proportional to the parameter N_o of the size distribution given by (1). Therefore, the ratio Z_e/R will depend on the characteristic size (D_m) and not on N_o . At the longer wavelength where non-Rayleigh effects are relatively small, Z_{eX}/R should be a monotonic function of D_m (subscript X here denotes the X-band radar reflectivity). This ratio could be approximated as a power-law function of D_m because Z_{eX} is proportional to the j th normalized moment of the size distribution (j is somewhat smaller than 6 because X-band scattering is not exactly in the Rayleigh regime) and R is proportional to its k th moment (k is usually somewhere between 3 and 4, depending on the density and fall velocity–size relationship), as can be seen from (5), (6), and (7):

$$Z_{eX}/R = AD_m^B. \quad (8)$$

b. Variability of Z_e – R – D_m relationships

Coefficients A and B in (8) depend on a priori assumptions about the snowflake density, the fall velocity–size relationship, the type of the snowflake size distribution, and the snowflake shape. However, the coefficients a and b in a single-wavelength Z_e – R relationship ($Z_e = aR^b$) depend on all of these assumptions [the sensitivity of Z_e – R relationships to snowflake density was considered by Matrosov (1992)] and also on the snowflake characteristic size, which is not available if radar measurements are taken only at one wavelength. The sensitivity of Z_e (and also a and b) to the characteristic size is the strongest among all the factors mentioned above. In the case of dual-wavelength measurements, the median size (D_m) is determined independently from dual-wavelength ratio measurements. Therefore, the variability in coefficients A and B from

one snowfall event to another should be much less than that in a and b , resulting in better snowfall-rate accuracies for dual-wavelength measurements compared to single-wavelength measurements.

Figure 3 shows model calculations of Z_{eX}/R as a function of D_m for different densities of dry snow (0.03, 0.05, and 0.07 g cm⁻³) and for fall velocity–size relationships, given by both (6) and (7). Spherical shapes were assumed in these calculations. The results were fitted by the power-law regressions. The best-fit values of coefficients A and B are given for each of the curves in this figure.

It can be seen from comparing curves for different densities (curves 1, 2, and 3 or curves 4, 5, and 6) that the snowflake density strongly influences Z_{eX} – R – D_m relationships. The variability of the coefficient B is rather small, while the variability of the coefficient A is significant. The choice of different fall velocity–size relationships also influences Z_{eX} – R – D_m relationships—however, at a lesser extent than density, especially for larger values of density ρ . Comparing curves 1 and 7 in Fig. 3 shows that the sensitivity of Z_{eX} – R – D_m relationships to the order of the assumed gamma size distribution is rather small. Calculations using the T-matrix approach for nonspherical particles indicate some sensitivity of A and B to snowflake shapes. This sensitivity, however, is not as strong as the one to density changes.

Generally, it could be considered that Z_{eX} – R – D_m relationships have four tuning parameters: the snowflake density ρ , the choice of the fall velocity–size relationship, the order of the gamma size distribution n , and the snowflake shape parameter (e.g., mean aspect ratio). However, because the sensitivity of these relationships to the density is the largest, it can be assumed that there is only one tuning parameter—the effective density, ρ_e . A value of ρ_e will also account for a priori assumptions about other influencing factors. This value can be tuned by matching snowfall accumulation results from simultaneous and collocated radar and ground snow-gauge measurements. For a further analysis, spherical shapes, the first-order gamma-function size distribution, and the fall velocity–snowflake-size relationship given by (7) are assumed.

In brief, the dual-wavelength method to measure snowfall rate can be formulated as follows: first the median snowflake size D_m is estimated from the DWR using (3) and then the snowfall rate R is estimated using D_m and the measurement of Z_{eX} from the Z_{eX} – R – D_m relationship (8) for a previously tuned value of the snowflake effective density ρ_e .

5. Experimental observations of snowfall using the dual-wavelength method

a. Description of the Snowrad experiment

A small, local experiment (hereafter referred to as Snowrad) to assess potentials of the dual-wavelength

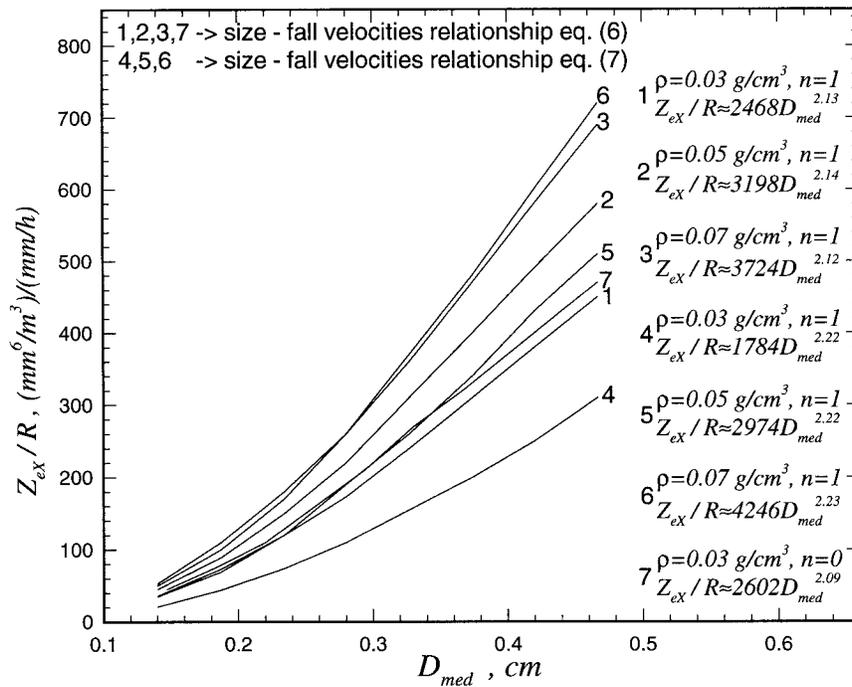


FIG. 3. The Z_e-R - D_m relationships at X band for different snowflake densities, fall velocity-size relationships, and orders of the gamma size distributions.

radar method for measuring snowfall was performed in January–March 1996 at the ETL experimental site near Boulder, Colorado. The site was equipped with instrumentation for ground snow measurements. The ETL X- and K_a -band radars were moved to the site for collocated and synchronous observations. The period of the Snowrad experiment was drier than usual, with only six measurable snowfall events during seven weeks. None of these events was heavy.

The main objectives of Snowrad were 1) to tune Z_{eX} - R - D_m relationships in terms of snowflake effective density ρ_e using snow gauges and flat surface measurements, 2) to assess the variability of the tuned values of ρ_e for different snowfall events, and 3) to compare results of the dual-wavelength method with snowfall estimates obtained from several existing Z_e - R relationships for the X band.

The ground snow instrumentation at the radar site included the standard, wind-shielded, heated precipitation gauge that measures total liquid accumulation with the 0.01" increments and three bucket-type precipitation gauges that were used for measurements of melted snow accumulation for specified time intervals. In addition, snowfall for the same intervals was measured by collecting accumulating snow from a flat surface that was cleaned after each accumulation measurement. Estimates of snow accumulation obtained by these different techniques were usually within 20%–25% for the same time periods.

Main characteristics of the NOAA X- and K_a -band radars are given by Kropfli et al. (1995). For most of

the time during snowfall events, both radars were in the fixed-beam mode with their beams oriented at an azimuthal angle of the horizontal wind v_g (in the upwind direction). The elevation angle β was chosen from the condition

$$\tan\beta = |w|/|v_g|, \tag{9}$$

where w is the Doppler velocity measured in the vertical direction. Typical values of β were about 10°–15° for most snowfall events. In this geometry, the radars were pointed toward incoming snowflakes that would eventually fall to the surface near the radar site. The fixed-beam mode was regularly interrupted by the sector, azimuthal (VAD), and RHI-type scans for estimating horizontal winds, vertical velocities of snowflakes, and the geometry of the snowstorm.

Special attention was paid to the best possible space collocation of radar measurements. The range resolution of both radars was 37.5 m. Of the most interest were radar measurements in the closest range gates that were free from ground clutter. The corresponding distances for such ranges were about 300–400 m, depending on the chosen azimuth and elevation. A small difference in antenna beamwidths (0.5° for the K_a -band radar and 0.8° for the X-band radar) was not crucial for the chosen close-range geometry. However, this issue could become important when observing regions with high gradients of snowfall parameters at longer ranges. The accuracy of the relative calibration of radar measurements was ensured by matching reflectivity data from both radars at the top of the radar echo (for the vertical

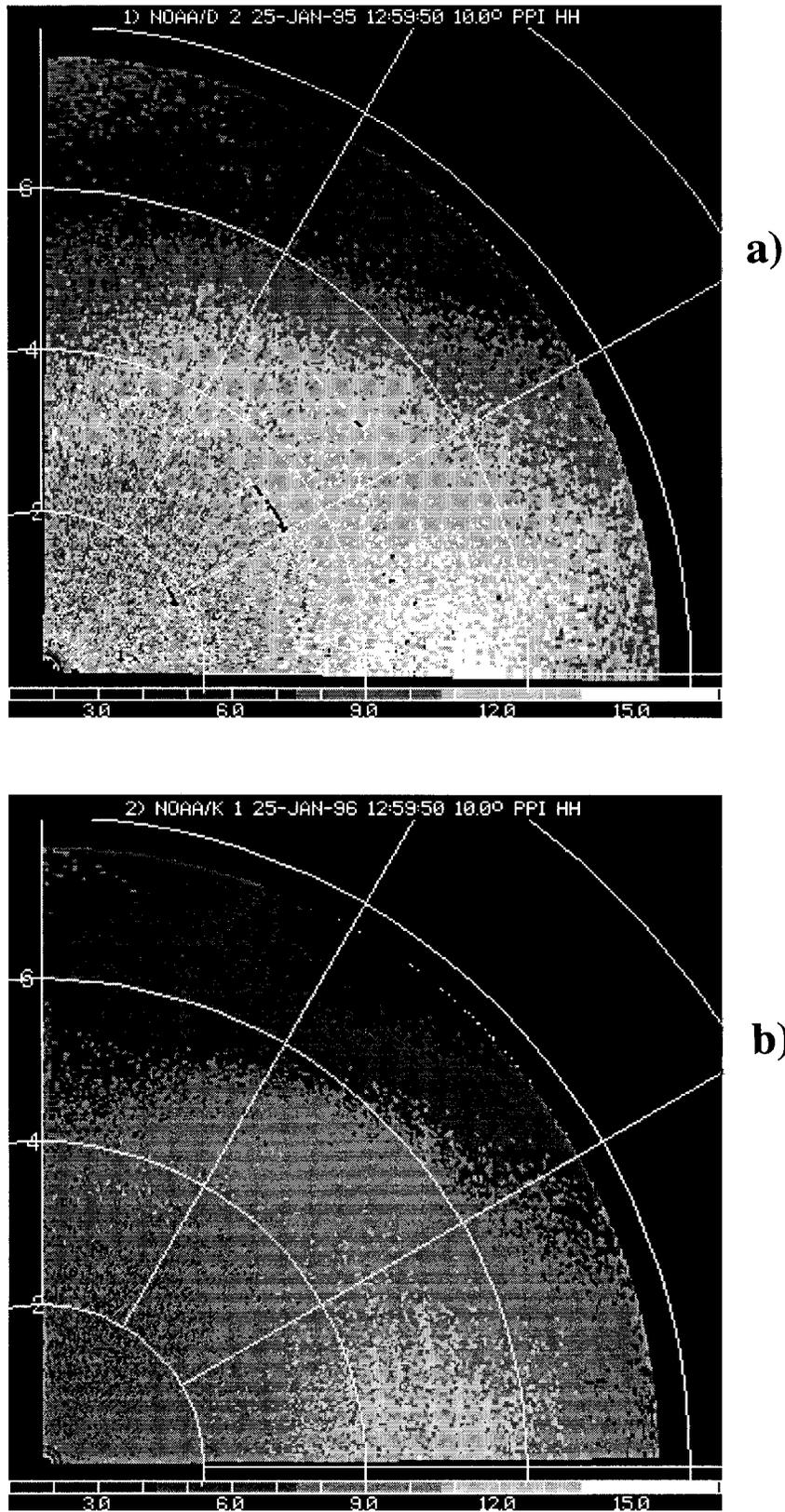


FIG. 4. Collocated and synchronized sector (0° - 90°) scans of the 25 January 1996 snowfall event at 1300 MST. (a) X band and (b) K_a band. The radar elevation angle is 10° .

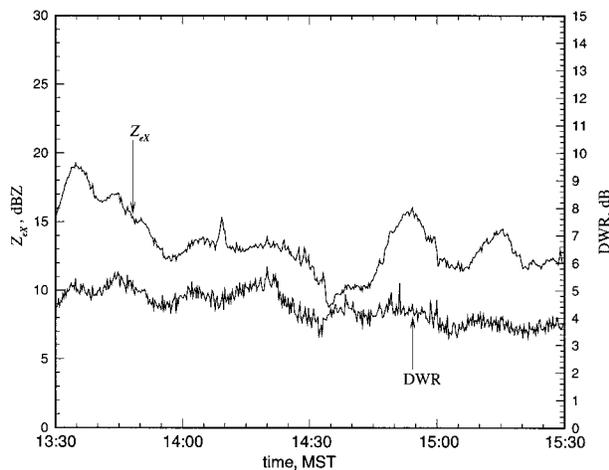


FIG. 5. Time series of the X-band reflectivity Z_{ex} and the dual-wavelength ratio DWR during the 25 January 1996 snowfall event.

beams), where the particles are the smallest and, therefore, DWR is close to 0.

b. Dual-wavelength data

Figure 4 shows two synchronized low-elevation ($\beta = 10^\circ$) sector scans of measured radar reflectivity during a snowstorm event observed on 25 January 1996. The surface air temperature during this event was about -4°C . There was no supercooled liquid water detected by microwave radiometer measurements. A similar snowstorm structure was observed by both radars; however, X-band reflectivities were significantly higher. Soon after these sector scans were taken, the radars were switched into the fixed-beam mode for 2 h of continuous observations at an azimuthal direction of 60° .

Time series of the X-band reflectivity Z_{ex} and DWR are shown in Fig. 5. The data represent mean reflectivity in three clutter-free range gates closest to the ground. Note that reflectivities and DWR values were nearly identical for all three of these range gates. The mean values of Z_{ex} and DWR for this observation period were 14 dBZ and 4.5 dB, respectively.

Melted snowfall accumulation as a function of time is presented in Fig. 6. Accumulation results are shown as obtained from the existing Z_{ex} - R relationships (Boucher and Wieler 1985; Fujiyoshi et al. 1990; Puhakka 1975), and from the dual-wavelengths method for two values of the effective snowflake density, $\rho_e = 0.03 \text{ g cm}^{-3}$ and 0.04 g cm^{-3} are shown. The average value of accumulation from snow-gauge and ground measurements for this 2-h period was 1.1 mm, shown by an arrow in Fig. 6. Here the dual-wavelength estimates of the 2-h snowfall accumulation are quite close to ground data if 0.03 g cm^{-3} is chosen for ρ_e . The existing Z_{ex} - R relationships significantly underestimate snow accumulation measured at the ground, and they differ from each other by as much as a factor of about 4. Part of

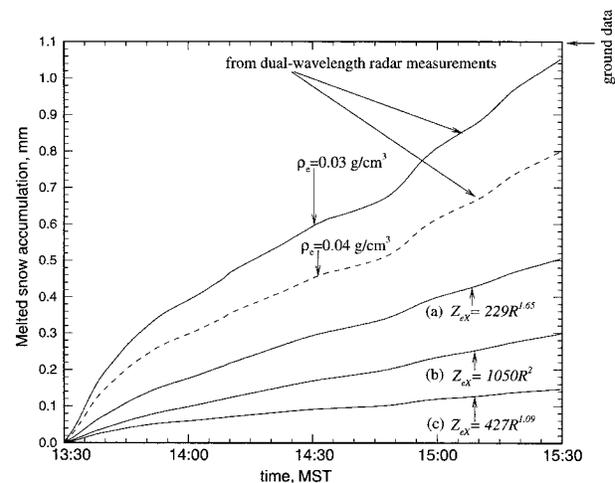


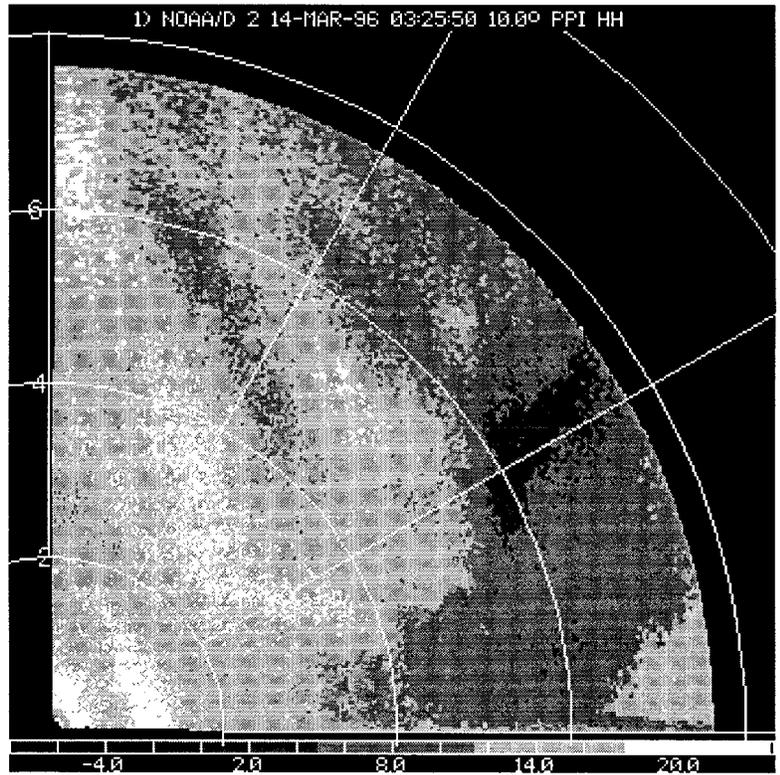
FIG. 6. Melted snow accumulation for the 25 January 1996 snowfall event from dual-wavelength radar measurements and from different existing single-parameter Z_{ex} - R relationships: (a) Boucher and Wieler (1985), (b) Puhakka (1975), and (c) Fujiyoshi et al. (1990).

the spread in accumulation values in Fig. 6 might be attributed to uncertainties in the absolute radar calibrations that are very difficult to achieve to better than about 1 dB.

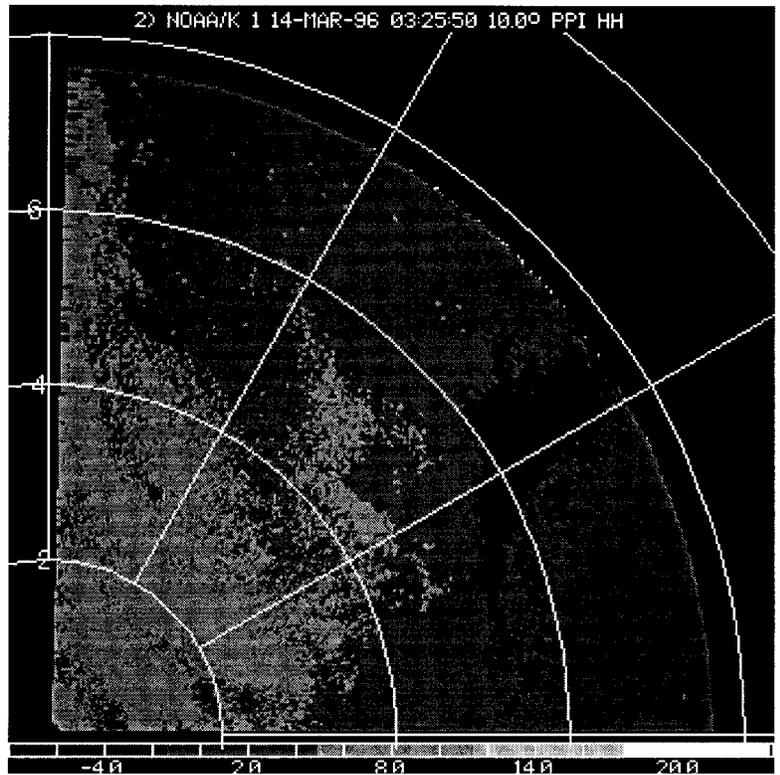
Another period of fixed-beam measurements, of about a 2-h duration, was conducted during a snowstorm on 14 March 1996. The surface air temperature in this case was close to the one during the previous event. Two synchronized sector scans ($\beta = 10^\circ$) performed just prior to the fixed-beam measurements are shown in Fig. 7. As in the previous case, patterns of the K_a - and X-band reflectivity fields are very similar. However DWR values are significantly higher than in the event of 25 January, indicating larger snowflakes. The size difference was difficult to express quantitatively because snowflake aggregates often break up on surface impact.

Figure 8 shows the 2-h time series of DWR and Z_{ex} during the fixed-beam measurements on 14 March 1996 at an azimuth of 30° . The mean X-band reflectivity and DWR during this period were 16.2 dBZ and 7.8 dB, respectively. As in the 25 January event (Fig. 6), Fig. 9 shows time series of melted snow accumulations as obtained using the existing empirical Z_{ex} - R relationships and the dual-wavelength method assuming $\rho_e = 0.03$ and 0.04 g cm^{-3} . The ground measurement of snowfall accumulation for this observational period was 0.75 mm (shown by an arrow).

It can be seen from Fig. 9 that an assumption that $\rho_e \approx 0.035 \text{ g cm}^{-3}$ provides the best agreement with the ground data. However, the dual-wavelength method result with $\rho_e = 0.03 \text{ g cm}^{-3}$ differs from the ground data by only about 13%, which is probably within the uncertainty of the ground accumulation measurements. The empirical relationship of Boucher and Wieler (1985) also gives a rather close approximation to the



a)



b)

FIG. 7. Same as Fig. 4 but for the 14 March 1996 snowfall event.

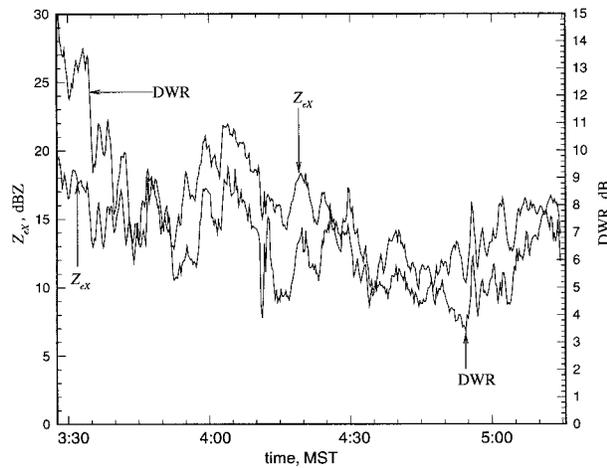


FIG. 8. Same as Fig. 5 but for the 14 March 1996 snowfall event.

ground measurement, while other empirical $Z_{eX}-R$ relationships significantly underestimate it.

It is worthwhile to compare the snowfall events of 25 January and 14 March in more detail. Table 1 lists parameters of these snowstorms. The 25 January storm was characterized by smaller snowflakes in greater number concentrations (compared with the snowfall of 14 March), which resulted in a larger accumulation for the same duration. Single-wavelength $Z_{eX}-R$ relationships incorrectly estimate accumulation for the 14 March event as greater than for the 25 January event by a factor of 1.3–1.7, depending on the value of the exponent b ($Z_{eX} = aR^b$), simply because X-band reflectivities were larger for the 14 March snowfall. This illustrates that single-wavelength relationships cannot distinguish between microphysical differences in snowfall.

Conversely, the dual-wavelength method captures the microphysical differences in these two events correctly due to independent estimates of snowflake characteristic sizes. This method yields the accumulation value for

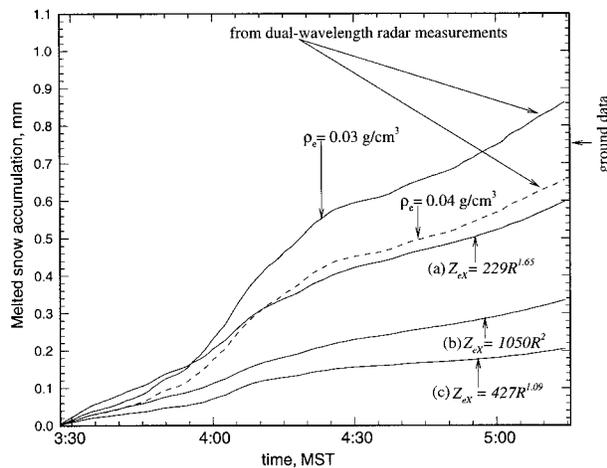


FIG. 9. Same as Fig. 6 but for the 14 March 1996 snowfall event.

TABLE 1. Parameters of two snowfall events observed on 25 January and 14 March 1996.

Event	Duration (h)	Mean Z_{eX} (dBZ)	Mean DWR (dB)	Melted accumulation (mm)
25 January 1996	2	14.0	4.5	1.1
14 March 1996	2	16.2	7.8	0.75

the 25 January snowfall greater than for the 14 March snowfall by about 20%, comparing well with the corresponding difference in melted snow accumulations from the ground measurements of about 30%.

The greatest DWR values measured during the Snowrad experiment were observed during the snowfall on 26 February 1996. Fixed-beam measurements during a 1.5-h period for this event are shown in Fig. 10. Maximum DWR values reached about 13 dB. This snowstorm was characterized by large aggregate snowflakes. Their number concentrations were, however, rather small, resulting in a low accumulation of only 0.13 mm.

The accumulation time series for the 26 February snowstorm are shown in Fig. 11. Note that the best fit for the dual-wavelength measurements is yielded by an assumption for the effective snowflake density $\rho_e \approx 0.04 \text{ g cm}^{-3}$. The Puhakka (1975) $Z_{eX}-R$ relationship provides a rather close approximation to the ground data for this case. The other two $Z_{eX}-R$ relationships, however, overestimate (Boucher and Wieler 1985) and underestimate (Fujiyoshi et al. 1990) the ground measurements by about a factor of 2.

c. Choice of the effective density tuning parameter

As was mentioned earlier, the proposed dual-wavelength method for measuring snowfall has one tuning parameter—effective snowflake density ρ_e . This param-

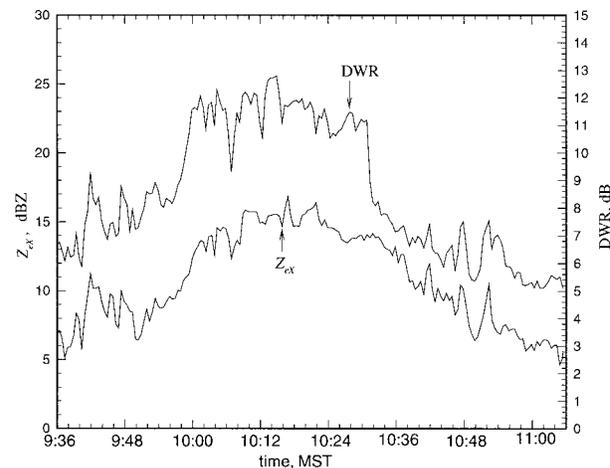


FIG. 10. Same as Fig. 5 but for the 26 February 1996 snowfall event.

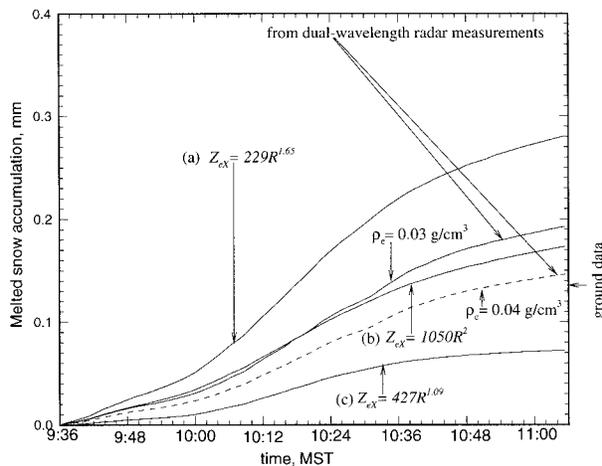


FIG. 11. Same as Fig. 6 but for the 26 February 1996 snowfall event.

eter accounts not only for the actual density of snowflakes but also for a priori assumptions about the snowflake-size-fall velocity relationship, the snowflake shape, and the size distribution type.

One of the purposes of the Snowrad experiment was to find best values of ρ_e for each snowstorm event by matching ground- and radar-derived melted snow accumulations and to estimate the variability of these values between different events. Analysis of all Snowrad data indicated that best values of ρ_e providing the match between ground- and radar-derived accumulations varied from 0.03 to 0.04 g cm⁻³. The dynamic range of these ρ_e changes corresponds to the assumptions of spherical particles, the first-order gamma-function size distribution, and the fall velocity-size relationship given by (7). Note that these assumptions stipulate the coefficients in Z_{ex} - R - D_m relationships used to derive the snowfall rate (see Fig. 3). From these three assumptions, ρ_e is most sensitive to the assumption about fall velocity-size relationship. A change of this relationship from (7) to (6), for example, results in a change of ρ_e by about 30%.

As one can see from Figs. 6, 9, and 11, differences in dual-wavelength estimates of accumulation for $\rho_e = 0.03$ g cm⁻³ and $\rho_e = 0.04$ g cm⁻³ are about 25%. Hence, a single choice of $\rho_e \approx 0.035$ g cm⁻³ provides a good agreement between radar and ground data. Based on the results of the Snowrad field experiment, this value of ρ_e can be used for dual-wavelength radar measurements of snowfall when no independent estimations of the tuning parameter are available.

The actual density of snowflakes ρ usually decreases with snowflake size (Magono and Nakamura 1965; Klaassen 1988). Therefore, one can expect that the tuning value of ρ_e should decrease when the characteristic size of snowflakes increases. The data from Snowrad were, however, too limited to determine a stable relationship between snowflake characteristic size and ρ_e

providing the match between ground and radar data. Note also that the effective density ρ_e accounts for other factors mentioned above and not only for the real snowflake density ρ , so one could expect lower variability of ρ_e with snowflake size compared with that of ρ . It should be admitted that a certain correlation between ρ_e and snowflake characteristic size could exist, but more field experiment data for a larger variety of conditions are needed to properly estimate this correlation. In future developments of the dual-wavelength radar method it could be more appropriate to choose a tuning value ρ_e based on the information on snowflake characteristic size that is estimated from DWR measurements [see (3)] rather than using $\rho_e = \text{const}$, regardless of the event.

6. Conclusions

Traditional, single-parameter Z_e - R relationships for obtaining snowfall rate R from radar reflectivity measurements alone exhibit a significant degree of variability in their coefficients, resulting in high uncertainties of snowfall estimates. One of the main reasons for this variability is a low correlation between characteristic size and concentration of snowflakes—the two parameters that to a major degree determine both the snowfall rate and its radar reflectivity. This makes single-parameter reflectivity measurements generally insufficient.

A dual-wavelength radar method was developed for estimating snowfall rate. The method implies simultaneous and collocated measurements at two wavelengths, one of which is in the Rayleigh scattering regime or at least not far from this regime for typical snowflake sizes. Scattering at the other wavelength should be substantially outside the Rayleigh regime. The dual-wavelength reflectivity ratio DWR at these two wavelengths is a function of the snowflake median size and does not depend significantly on snowflake density and details of their size distribution. This function can be approximated by the power law for DWR ≤ 15 dB. Note that DWR, if expressed as a function of snowflake characteristic sizes other than median, does depend on details of the size distribution.

After the median snowflake size D_m is determined from the dual-wavelength ratio measurements, the snowfall rate can be estimated from Z_e - R - D_m relationships, where Z_e is the reflectivity at the longer wavelength. These relationships are constructed in the form $Z_e/R = AD_m^B$, and they depend on the snowflake density, shape, details of the size distribution, and the relation between snowflake sizes and fall velocities. The dependence on density is, however, the strongest. This fact allows one to use only a single tuning parameter—the effective snowflake density, ρ_e , for these relationships. A tuned value of ρ_e also accounts for a priori assumptions about other less influential parameters.

A field experiment was conducted near Boulder, Colorado, using NOAA's K_a- and X-band radars and col-

located ground snow measurements. The ground measurements were used to find values for the tuning parameter ρ_e . Experimental data indicated that the dual-wavelength method provides a good match for ground snow measurements if ρ_e is between 0.03 and 0.04 g cm⁻³, depending on the snowfall event. The difference in the dual-wavelength snow accumulation estimates for assumptions 0.03 and 0.04 g cm⁻³ is about 25%, while existing X-band Z_e - R relationships produced accumulations that differ from each other by as much as a factor of 4 for the same dataset. More experimental data are needed to better determine the range of applicability of these ρ_e assumptions and, possibly, to introduce a median-size dependent tuning value of ρ_e .

An independent estimate of snowflake characteristic size in the dual-wavelength approach accounts for much of the improvement of snowfall estimates over a single-parameter approach. This estimate allows one to resolve the ambiguity between contributions of snowflake sizes and their concentration to snowfall rate and reflectivity and, thus, to distinguish between microphysically different events.

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REFERENCES

- Atlas, D., and A. C. Chmela, 1957: Physical-synoptic variations of raindrop size parameters. *Proc. Sixth Weather Radar Conference*, Cambridge, MA, Amer. Meteor. Soc., 21-29.
- , M. Kerker, and W. Hitschfeld, 1953: Scattering and attenuation by non-spherical atmospheric particles. *J. Atmos. Terr. Phys.*, **3**, 108-119.
- Barber, P., and C. Yeh, 1975: Scattering of electromagnetic waves by arbitrarily shaped dielectric bodies. *Appl. Opt.*, **14**, 2864-2872.
- Battan, L. J., 1973: *Radar Observations of the Atmosphere*. University of Chicago Press, 324 pp.
- Boucher, R. J., and J. G. Wieler, 1985: Radar determination of snowfall rate and accumulation. *J. Climate Appl. Meteor.*, **24**, 68-73.
- Braham, R. R., Jr., 1990: Snow particle size spectra in lake effect snows. *J. Appl. Meteor.*, **29**, 200-207.
- Fujiyoshi, Y., T. Endoh, T. Yamada, K. Tsuboki, Y. Tachibana, and G. Wakahama, 1990: Determination of a Z - R relationship for snowfall using a radar and high sensitivity snow gauges. *J. Appl. Meteor.*, **29**, 147-152.
- Gunn, K. L. S., and J. S. Marshall, 1958: The distribution with size of aggregate snowflakes. *J. Meteor.*, **15**, 452-461.
- Ihara, T., Y. Furuhashi, and K. Tohma, 1982: Measurement of depolarization due to snowfall at 34.5 GHz. *Trans. Inst. Electron. Commun. Eng. Japan*, **65E**, 16-22.
- Illingworth, A. J., J. W. G. Thomason, J. W. F. Goddard, H. Sauvegeot, and P. Simpson, 1995: Sizing of ice aggregates and snowflakes using coincident 3 and 35 GHz reflectivity observations. Preprints, *27th Conf. on Radar Meteorology*, Vail, CO, American Meteor. Soc., 50-52.
- Klaassen, W., 1988: Radar observations and simulation of the melting layer of precipitation. *J. Atmos. Sci.*, **45**, 3741-3753.
- Kropfli, R. A., and Coauthors, 1995: Cloud physics studies with 8 mm wavelength radar. *Atmos. Res.*, **35**, 299-313.
- Magono, C., and T. Nakamura, 1965: Aerodynamic study of falling snowflakes. *J. Meteor. Soc. Japan*, **43**, 139-143.
- Matrosov, S. Y., 1992: Radar reflectivity in snowfall. *IEEE Trans. Geosci. Remote Sens.*, **30**, 454-461.
- , 1993: Possibilities of cirrus particle sizing from dual-frequency radar measurements. *J. Geophys. Res.*, **98**, 20 675-20 683.
- , R. F. Reinking, R. A. Kropfli, and B. W. Bartram 1996: Estimation of ice hydrometeor types and shapes from radar polarization measurements. *J. Atmos. Oceanic Technol.*, **13**, 85-96.
- Mitchell, D. L., 1996: Use of mass- and area-dimensional power laws for determining precipitation particle fall velocities. *J. Atmos. Sci.*, **53**, 1710-1723.
- Puhakka, T., 1975: On the dependence of Z - R relation on the temperature in snowfall. Preprints, *16th Radar Meteorology Conference*, Houston, TX, Amer. Meteor. Soc., 504-507.
- Sekhon, R. S., and R. C. Srivastava, 1970: Snow size spectra and radar reflectivity. *J. Atmos. Sci.*, **27**, 299-307.